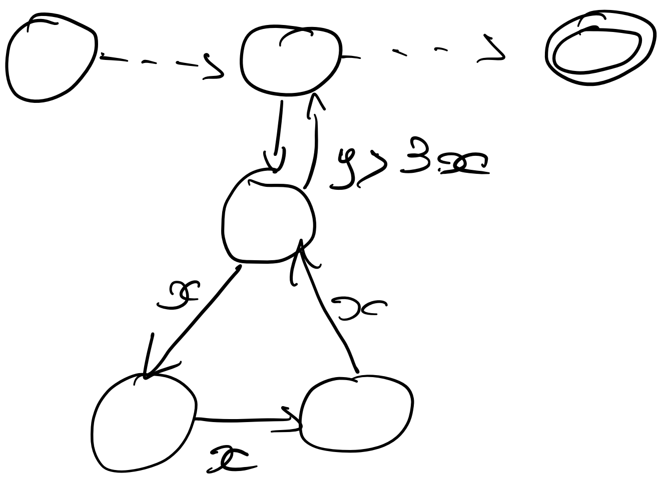
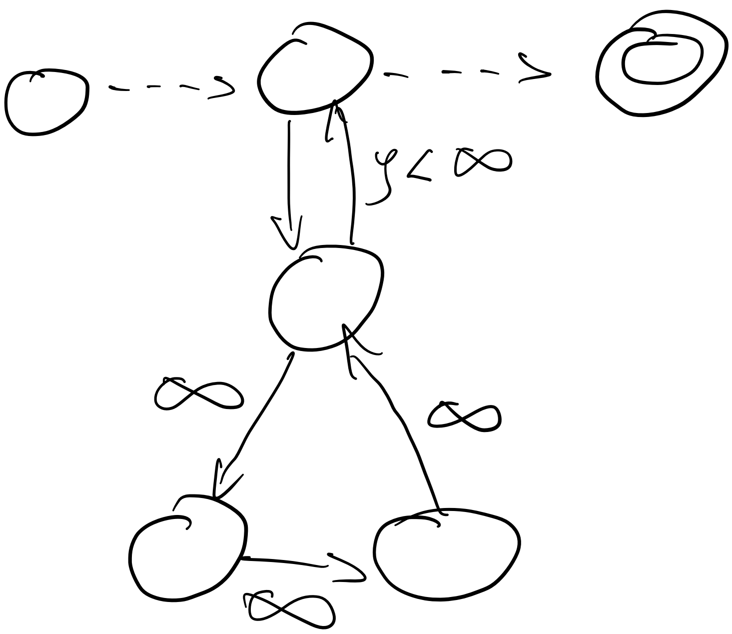
Screen Shot 2021-07-22 at 11.50.42 AM

Proof by contradiction :



$t \to \infty$



Proof by induction:

- The h-values are admissible at time t. Thus they are bounded from above by the goal distances which are finite since the state space is safely explorable.

- For $t=0$, the execution cost (and upper bound) are 0.

- We assume it's true for time t.

- For $t+1$:
  - The execution cost increases by $w(v^t, a^{t+1})$
  - The upper bound increase by:

$$
\begin{aligned}
&\sum_{u\in S\setminus\{u^{t+1}\}} h^{t+1}(u) - \sum_{s\in S\setminus\{u^t\}} h^t(u) \\
&= \sum_{u\in S\setminus\{u^t,u^{t+1}\}} [h^{t+1}(u)-h^t(u)] + h^{t+1}(u^t) - h^t(u^{t+1}) \\
&\overset{11.1}{=} \sum_{u\in S\setminus\{u^t,u^{t+1}\}} [h^{t+1}(u)-h^t(u)] + \max\{h^t(u^t), \min_{a\in A(u^t)}\{w(u^t,a)+h^{t+1}(Succ(u^t,a))\}\} - h^t(u^{t+1}) \\
&\geq \sum_{u\in S\setminus\{u^t,u^{t+1}\}} [h^{t+1}(u)-h^t(u)] + \min_{a\in A(u^t)}\{w(u^t,a)+h^{t+1}(Succ(u^t,a))\} - h^t(u^{t+1}) \\
&\geq \sum_{u\in S\setminus\{u^t,u^{t+1}\}} [h^{t+1}(u)-h^t(u)] + w(u^t,a^{t+1}) + h^{t+1}(u^{t+1}) - h^t(u^{t+1}) \\
&= \sum_{u\in S\setminus\{u^t\}} [h^{t+1}(u)-h^t(u)] + w(u^t,a^{t+1}) \\
&\overset{11.2}{\geq} w(u^t,a^{t+1}),
\end{aligned}
$$

Lemma

Completness of LRTA*:

LRTA* with admissible initial h-values h^0 reaches a goal state with execution cost of at most $h^0(s) + \sum_{v\in S}[\delta(v,T) - h^0(v)]$

Proof according to Lemma

(at most $\sum_{v\in S}[h^t(v) - h^0(v)] - (h^t(v^t) - h^0(v^0))$)

$$\sum_{u \in S} [h^t(u) - h^0(u)] - (h^t(s^t) - h^0(s^0)) \leq$$

$$\sum_{u \in S} [d(u,T) - h^0(u)] + h^0(s^0)$$

$$= h^0(s) + \sum_{u \in S} [\delta(u,T) - h^0(u)].$$

Admissible initial h-values (that is h-values that are lower bounds on the corresponding goal distances) remain admissible after every value-update step of LRTA* and are monotonically nondecreasing.

Similarly, consistent initial h-values (i.e., initial h-values that satisfy the triangle inequality) remain consistent after every value-update step of LRTA* and are monotonically nondecreasing.